Natural Deduction Rules with Iterated Quantifiers

G. J. Mattey

Winter, 2009 / Philosophy 112

Iterated Universal Quantifiers

- There are no restrictions on instantiating universal sentences, which means there is no problem with iterated universal quantifiers.
- For example, '(∀x)(∀y)Fxy' can be instantiated as:
  - (∀y)Fay, which in turn can be instantiated as:
    - Faa, or
    - Fab, etc.

Iterated Existential Quantifiers

- The instantiation of existential quantifiers is subject to restrictions which rule out some instantiations in the case of multiple quantifiers.
- For example, the following pattern of instantiation is *not* permitted:

1	(∃x)(∃y)Fxy	P
2	a (∃y)Fay	A
3	a Faa	A
4		

- The second instantiation violates the restriction that the instantiating name be isolated in the derivation.

Hard Problems

- On page 94, a derivation in 21 steps is given.
- We are told what the two main strategic moves are, but not why they were chosen.
- Reductio is chosen as the basic strategy.
  - The reason is that it would be impossible to use ∃ I to get the conclusion.

  - This condition usually holds, and so reductio is a good strategy for deriving existential sentences.

- The sub-strategy is to use reductio again to get 'Pâ' in order to use $\forall$ I.
  - Once again, it is impossible to get this result in any other way.
  - This condition usually holds, so reductio is a good strategy for deriving atomic sentences that cannot be derived using instantiation.