

Solutions to Exercises for Functions and Identity

January 25, 2005

1 Exercises FI-1

- a) No. There is only a constant term, which is not attached to a predicate.
- b) No. One cannot negate a constant term.
- c) Yes. Note that the negation sign governs the whole sentence, whose predicate is the identity symbol.
- d) Yes.
- e) No. The outer function symbol is not properly filled in, because the function symbol inside it is not filled in.
- f) No. There is no function symbol to go with the parentheses. The 'f' would have to be italicized.
- g) No. Only a constant term can flank an identity sign, and ' $Bf(a)$ ' is not a constant term.
- h) Yes.

2 Exercises FI-2

- a) This says that two is greater than the successor of 1, which is false.
- b) This says that $1 + 2 = 3$, which is true.
- c) This says that the successor of the successor of 1 is identical to the successor of 2. The successor of 1 is two, and the successor of two is 3, so the successor of the successor of 1 is 3. $3 = 3$, so the sentence is true.
- d) This says that the successor of 2 is identical to the result of adding 1 to 2. The successor of 2 is 3, as is the result of adding 1 to 2, the sentence is true.
- e) This says that the result of adding the successor of 1 to the successor of 1 is greater than two. The successor of 1 is 2, and the result of adding 2 to 2 is 4, which is greater than 2. So the sentence is true.

3 Exercises FI-3

- a) $Gf(o,t)f(o,w)$
- b) $\sim Gs(o)s(w)$

c) $Gs(o) \vee s(o) = o$

4 Exercises FI-4

a)

1	a = b	P
2	b = c	P
3	a = c	1 2 = E

b)

1	a = b	P
2	Ka	A
3	Kb	1 2 = E
4	Kb	A
5	Ka	1 4 = E
6	Ka \equiv Kb	2-3 4-5 \equiv I

c)

1	Lab	P
2	b = f(a)	P
3	c = f(b)	P
4	Lf(b)c	P
5	Laf(a)	1 2 = E
6	Lcc	3 4 = E
7	Laf(a) & Lcc	5 6 & I

d)

1	~ a = c	P																								
2	~ (~ a = b ∨ ~ b = c)	A																								
3	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;">3</td> <td style="width: 35%; border-left: 1px solid black; padding-left: 5px;">a = b</td> <td style="width: 60%; text-align: right; vertical-align: top;">A</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">4</td> <td style="border-left: 1px solid black; padding-left: 5px;">~ b = c</td> <td style="text-align: right; vertical-align: top;">1 3 = E</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">5</td> <td style="border-left: 1px solid black; padding-left: 5px;">~ a = b ∨ ~ b = c</td> <td style="text-align: right; vertical-align: top;">4 ∨ I</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">6</td> <td style="border-left: 1px solid black; padding-left: 5px;">~ (~ a = b ∨ ~ b = c)</td> <td style="text-align: right; vertical-align: top;">2 R</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">7</td> <td style="border-left: 1px solid black; padding-left: 5px;">~ a = b</td> <td style="text-align: right; vertical-align: top;">3-6 ~ I</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">8</td> <td style="border-left: 1px solid black; padding-left: 5px;">~ a = b ∨ ~ b = c</td> <td style="text-align: right; vertical-align: top;">7 ∨ I</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">9</td> <td style="border-left: 1px solid black; padding-left: 5px;">~~ (~ a = b ∨ ~ b = c)</td> <td style="text-align: right; vertical-align: top;">2-8 ~ I</td> </tr> <tr> <td style="text-align: right; vertical-align: top;">10</td> <td style="border-left: 1px solid black; padding-left: 5px;">~ a = b ∨ ~ b = c</td> <td style="text-align: right; vertical-align: top;">9 ~ E</td> </tr> </table>	3	a = b	A	4	~ b = c	1 3 = E	5	~ a = b ∨ ~ b = c	4 ∨ I	6	~ (~ a = b ∨ ~ b = c)	2 R	7	~ a = b	3-6 ~ I	8	~ a = b ∨ ~ b = c	7 ∨ I	9	~~ (~ a = b ∨ ~ b = c)	2-8 ~ I	10	~ a = b ∨ ~ b = c	9 ~ E	
3	a = b	A																								
4	~ b = c	1 3 = E																								
5	~ a = b ∨ ~ b = c	4 ∨ I																								
6	~ (~ a = b ∨ ~ b = c)	2 R																								
7	~ a = b	3-6 ~ I																								
8	~ a = b ∨ ~ b = c	7 ∨ I																								
9	~~ (~ a = b ∨ ~ b = c)	2-8 ~ I																								
10	~ a = b ∨ ~ b = c	9 ~ E																								

5 Exercises FI-5

Suppose 'Lea' and 'a = e' are true in an interpretation. Then 'a' and 'e' designate the same member of the domain of that interpretation. Since 'Lea' is true in the interpretation, the ordered pair consisting of what 'a' designates followed by what 'e' designates is in the extension of 'L'. Therefore, the ordered pair consisting of what 'a' designates followed by what 'a' designates is in the extension of 'L', as is the ordered pair consisting of what 'e' designates followed by what 'e' designates. Thus, 'Laa' and 'Lee' are true on that interpretation, which was to be proved.