

Exercises for Functions and Identity

January 14, 2009

Due in class: January 20, 2009

1 Exercise 1

State whether or not the following expressions are terms or sentences of Predicate Logic. If the expression is a term or sentence of Predicate Logic, state which one it is. If it is not a term or expression of Predicate Logic, state why not.

- a) $f(ba)$
- b) $f(a) = a$
- c) $f(a,b) = g(a)$
- d) $Gf(a,b,c)d f(a)$
- e) $f \sim (f)a$
- f) $gf(a)$
- g) $Cb = f(a)$
- h) $f(Lf(a))$

2 Exercise 2

Given the following transcription guide, determine whether the following sentences are true or false. State how you determined that the sentence is true or that it is false.

D = the set of all positive integers: 1, 2, 3, ...

o: 1

w: 2

t: 3

f : the multiplication function

s : the successor or "next number" function [e.g., the successor of 1 is 2]

Lxy: x is less than y

- a) $Lts(w)$
- b) $f(o,w) = s(o)$

- c) $s(s(s(w))) = f(w,w)$
- d) $f(t,o) = t$
- e) $Lf(w,o)f(o,o)$

3 Exercise 3

Using the above transcription guide, transcribe the following sentences into Predicate Logic.

- a) $(2 \times 3) < (2 \times (2 \times 2))$.
- b) The successor of (2×1) is not less than than the successor of 1.
- c) 1 is less than or equal to the successor of 1.

4 Exercise 4

Show that the following derivability claims hold, using Sentence Logic rules plus the introduction and elimination rules for '='.

- a) $\{a = b\} \vdash b = a$
- b) $\{a = b, f(a) = c, c = d\} \vdash f(b) = d$
- c) $\{Lf(a)f(b), a = f(b)\} \vdash Lf(f(b))a$
- d) $\{\sim a = c\} \vdash \sim c = a$

5 Exercise 5

Show that the following property [an instance of "The Indiscernibility of Identicals"] holds in the semantics for Predicate Logic. Assume the antecedent of the outer conditional and give an argument for why the consequent conditional follows. [Hint: assume the antecedent of the consequent conditional and show why its consequent holds.]

If 'a = e' is true in an interpretation, then if 'La' is true in that interpretation, then 'Le' is true in that interpretation.