

Formal Semantics for Atomic Sentences

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Formal Semantics for Atomic Sentences

Interpretations

- In giving the formal semantics for atomic sentences of Predicate Logic, we must introduce a number of new metavariables.
 - ‘**I**’ designates an interpretation.
 - ‘**D**’ designates a set containing at least one object (the domain of the interpretation).
 - ‘**v**’ designates a (“valuation”) function which gives the designation of constant terms, the extensions of predicates, and the truth-values of sentence letters.
 - ‘**f**’, ‘**g**’, ‘**h**’ designate function symbols.
- **I** is an ordered pair $\langle \mathbf{D}, \mathbf{v} \rangle$.

Specifying the Domain

- We may specify the domain by enumerating its members:
 - $\mathbf{D} = \{ \text{Adam, Eve} \}$
 - $\mathbf{D} = \{ 1, 2, 3, 4, 5 \}$

A domain may also be specified by description:

- $\mathbf{D} =$ the set of all positive integers
- $\mathbf{D} = \{ x: x \text{ is a positive integer} \}$

More generally, we may refer to objects in the domain using the metavariables $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \dots$

The Valuation Function

- In the following interpretation $I = \{\mathbf{D}, \mathbf{v}\}$, the valuation function 'v' assigns truth-values to sentence letters.

- $v(P) = t$
- $v(Q) = f$

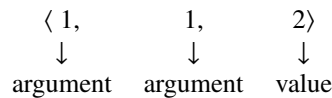
v assigns members of the domain to names.

- $v(a) = \text{Adam}$
- $v(e) = \text{Eve}$

- v assigns extensions to predicates.
 - $v(B) = \{\langle \text{Adam} \rangle\}$
 - $v(B) = \{x: x \text{ is blond}\}$
 - $v(L) = \{\langle \text{Adam}, \text{Eve} \rangle, \langle \text{Eve}, \text{Adam} \rangle\}$
 - $v(L) = \{\langle x, y \rangle: x \text{ loves } y\}$

Valuations of Function Symbols

- The valuation function assigns a value to each function symbol (without parentheses).
- For a function symbol of n places, the value is a set of n+1-tuples.
 - Consider the two-place addition function and a domain consisting of the positive integers.
 - $v(f) = \{\langle 1, 1, 2 \rangle, \langle 1, 2, 3 \rangle, \langle 1, 3, 4 \rangle, \dots\}$.
- The first n places specify the values of the argument terms.
- The last place specifies the value of the function.



Values of Filled-in Function Symbols

- The value assigned to the function symbol allows the determination of the value of a filled-in symbol.
- Let f be a two place function.
- Then $v(f)$ is a set of ordered triples.

- $v(f(t, u))$ is the last member of the the ordered triple whose first two members are $v(t)$ and $v(u)$.
 - Let $v(f)$ be the addition function as specified above.
 - Let $v(a) = 1$.
 - Then $v(f(a,a))$ is the last member of $\langle v(a), v(a), 2 \rangle$, which is the same as $\langle 1, 1, 2 \rangle$, and therefore is 2.

Symbols for Set Membership and Non-Membership

- It is convenient to have a symbol indicating membership in a set.
- The symbol ‘ \in ’ indicates set-membership.
 - Adam \in {Adam, Eve}
- The membership or “belongs to” symbol when struck through indicates non-membership.
 - Cid \notin {Adam, Eve}

Truth-Definition for Atomic Sentences

- Let P be an n-place predicate.
- Let t_1, t_2, \dots, t_n be constant terms.
- Let $Pt_1t_2 \dots t_n$ be an atomic sentence of Predicate Logic consisting of the n-place predicate P followed by n constant terms t_1, t_2, \dots, t_n , in that order.

<p>Truth-Definition for Atomic Sentences of Predicate Logic</p> <p>$v(Pt_1t_2 \dots t_n) = t$ if $\langle v(t_1), v(t_2) \dots v(t_n) \rangle \in v(P)$</p> <p>$v(Pt_1t_2 \dots t_n) = f$ otherwise</p>

- Compare this definition with the informal definition given earlier.
- An atomic sentence with an n-place predicate is true in an interpretation if the n-tuple consisting of the referents of the arguments (from left to right) is a member of the extension of the predicate; the sentence is false otherwise.

An Example

- Let $I = \{D, v\}$.
- $D = \{Adam, Eve\}$
- $v(a) = Adam$

- $v(e) = \text{Eve}$
- $v(L) = \{\langle \text{Adam}, \text{Eve} \rangle, \langle \text{Eve}, \text{Adam} \rangle\}$
- So, $\langle \text{Adam}, \text{Eve} \rangle \in v(L)$
- So, $\langle v(a), v(e) \rangle \in v(L)$
- So, 'Lae' is true in **I**.

Another Example

- Let $\mathbf{I} = \{\mathbf{D}, \mathbf{v}\}$.
- \mathbf{D} = the set of all positive integers.
- $v(o) = 1$
- $v(t) = 2$
- $v(f) = \{\langle 1, 1, 2 \rangle, \langle 1, 2, 3 \rangle, \dots\}$ [the addition function]
- $v(G) = \{\langle x, y \rangle \mid x > y\}$
- So, $v(f(o, o)) = 2$ [$1 + 1 = 2$]
- So, $\langle 2, 2 \rangle \notin v(G)$
- So, $\langle v(t), v(f(o, o)) \rangle \notin v(G)$
- So, 'Gtf(o, o)' is false in **I**.