Definite Descriptions

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Lower and Upper Bounds

- We can set lower and upper bounds on the number of objects which a sentence of predicate logic asserts as having a certain property or standing in a certain relation.
  - At least one, at least two, at least three, . . .
  - At most one, at most two, at most three, . . .
  - Exactly one, exactly two, exactly three, ...
- These bounds can be specified by the use of multiple quantifiers and the identity symbol (except for the case of "at least one").

Lower Bounds

- To say there are at least n objects with a certain property, one needs to use n existential quantifiers and state (if more than one existential quantifier is used) non-identities for all the distinct variables they contain.
  - There is at least one P: $(\exists x)Px$
  - There are at least two Ps: $(\exists x)(\exists y)((Px \& Py) \& x \neq y)$.
  - There are at least three Ps: $(\exists x)(\exists y)(\exists z)[(Px \& Py \& Pz) \& (x \neq y \& y \neq z \& x \neq z)]$.

Upper Bounds

- To say there are at most n objects with a certain property, one needs to use $n+1$ universal quantifiers and state identity for at least one pair of distinct variables the quantifiers contain.
  - There is at most one P: $(\forall x)(\forall y)((Px \& Py) \supset x = y)$.
  - There are at most two Ps: $(\forall x)(\forall y)(\forall z)[(Px \& Py \& Pz) \supset (x = y \vee y = z \vee x = z)]$.
  - There are at most three Ps: $(\forall x)(\forall y)(\forall z)(\forall w)[(Px \& Py \& Pz \& Pw) \supset (x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w)]$.

Exact Quantities

- To say that there are exactly *n* objects with a certain property, one must state that there are at least *n* and at most *n* such objects.
- It is possible to combine these specifications in a single sentence.
  - Exactly one P: (∃x)(Px & (∀y)(Py ⊃ x = y)).
  - Exactly two Ps: (∃x)(∃y)[((Px & Py) & x ≠ y) & (∀z)(Pz ⊃ (x = z ∨ y = z))].
- In general, *n* existential quantifiers are combined with a single universal quantifier.

E Shriek

- It is traditional for logicians to use a special quantifier symbol, '∃!,' pronounced "E shriek," to indicate that exactly one object has the given property.
- The exclamation point can indicate shrieking, from which the name may have been taken.
- Note that the text places the '!' after the variable in a quantifier, which is contrary to common usage and therefore not adopted here.
- There is exactly one P: (∃x)(Px & (∀y)(Py ⊃ x = y)). (∃!x)Px.
- For any open sentence **P**(**u**) with **u** a free variable, (∃!**u**)**P**(**u**) is shorthand for (∃**u**)[**P**(**u**) & (∀**v**)(**P**(**v**) ⊃ **v** = **u**)], where **v** is free for **u** in **P**(**u**) [i.e., **v** is free at all the places where **u** is free in **P**(**u**)].

Definite Descriptions

- Grammatically, a **definite description** is a term (italicized in the examples below) which can serve as a subject of a predicate or as a term in a relation.
  - *The first person to cross the finish line* wins the race.
  - *The tallest member of the Houston Rockets* is taller than *the tallest member of the Golden State Warriors*.
- Semantically, a definite description is supposed to pick out exactly one object, the one and only one that falls under the description it contains.
- In the second sentence, on February 13, 2009, the first definite description refers to Yao Ming and the second to Andris Biedrins.
- For a sentence containing a definite description to be true, there must be an object in the domain falling under the description (the existence condition) and there must be only one (the uniqueness condition).

Non-Denoting Definite Descriptions

- In 1905, Bertrand Russell raised the question of the truth-value of sentences containing non-denoting definite descriptions ("On Denoting").
  - *The present king of France* is bald.
- There is no present king of France, so the definite description does not denote.
- Russell decided to treat any such sentence as false, on the grounds that the use of the definite description in the sentence implies the existence of what the description ostensibly denotes.
- Thus, the sentence '*The present king of France* is bald' is false.
- An alternative approach is to declare that the sentence has no truth-value when a definite description it contains fails to denote.

Symbolizing Definite Descriptions

- Russell held that definite descriptions should be symbolized **contextually**.
- That is, there is no special symbol for the description itself, but any sentence containing a definite description can be symbolized by other means.
- Teller's (corrected) way of doing this is to write that there is exactly one thing satisfying the condition of the description, and that any such thing satisfies the predicate of the sentence itself.
  - Fx: x is present king of France, Bx: x is bald.
  - $(\exists !x)Fx \,\&\, (\forall x)(Fx \supset Bx)$.
  - $(\exists x)(Fx \,\&\, (\forall y)(Fy \supset x = y)) \,\&\, (\forall x)(Fx \supset Bx)$.
- Russell himself symbolized it in the following (equivalent) way:
  - $(\exists x)(Fx \,\&\, (\forall y)(Fy \supset x = y) \,\&\, Bx)$

Syntactical Proof of Implication of Russell's Transcription by Teller's

1	$(\exists x)(Fx \,\&\, (\forall y)(Fy \supset x = y)) \,\&\, (\forall x)(Fx \supset Bx)$	P
2	$(\forall x)(Fx \supset Bx)$	1 & E
3	$(\exists x)(Fx \,\&\, (\forall y)(Fy \supset x = y))$	1 & E
4	a $\quad Fa \,\&\, (\forall y)(Fy \supset a = y))$	A
5	$\quad Fa \supset Ba$	2 ∀ E
6	$\quad Fa$	4 & E
7	$\quad Ba$	5 6 ⊃ E
8	$\quad Fa \,\&\, (\forall y)(Fy \supset a = y) \,\&\, Ba$	4 7 & I
9	$\quad (\exists x)(Fx \,\&\, (\forall y)(Fy \supset x = y) \,\&\, Bx)$	8 ∃ I
10	$(\exists x)(Fx \,\&\, (\forall y)(Fy \supset x = y) \,\&\, Bx)$	1 4-9 ∃ E

An Erroneous Formulation

- The example (1a) for symbolizing definite descriptions in *A Modern Formal Language Primer* Volume II, p. 153, is incorrect.
- The first symbolization in the earlier slide is a correction of this error.
- The erroneous formulation of "the F is B" is:
  - (∃!x)(Fx & Bx), which should be rewritten as:
  - (∃x)[(Fx & Bx) & (∀y)((Fy & By) ⊃ x = y)], and
- The mistaken formulation would be equivalent to the Russell formulation, but for the occurrence of 'By' in the antecedent of the conditional.
- The error would result in false sentences being counted as true.

A Counterexample

- Here is a semantical counterexample to the mistaken formulation.
- In the interpretation, exactly one object in the domain satisfies both the description and the predicate of the sentence, but more than one object satisfies the description itself.
- Consider a domain with two objects, the number 1 and the number 3.
- The description 'the odd number' would fail to denote given such a domain, since more than one object satisfies the description 'odd number'.

- However, 'The odd number is less than two' is true on the formulation, where Ox: x is odd, Tx: x is less than two.
- $(\exists x)[(Ox \,\&\, Tx) \,\&\, (\forall y)((Oy \,\&\, Ty) \supset x = y)]$.

Another Formalization of Definite Descriptions

- Since 'the . . .' functions as a term in English, we might wish to represent it as a kind of quantifier in Predicate Logic.
- Prefixing this quantifier to an open sentence containing its variable would yield a term.
- The descriptive part of the description is written as an open sentence **P**(**u**).
- This is prefixed by '(The **u**),' to form the term (The **u**)**P**(**u**).
  - 'The odd number' is symbolized as '(The x)Ox.' 'The odd number is less than two' would then be 'T(The x)Ox.'
- Generally, where **P**(**u**) and **Q**(**u**) are open sentences with **u** as the only free variable, **Q**[(The **u**)**Pu**] is shorthand for: $(\exists!\mathbf{u})[\mathbf{P}(\mathbf{u}) \,\&\, (\forall \mathbf{u})(\mathbf{P}(\mathbf{u}) \supset \mathbf{Q}(\mathbf{u})]$.

Wide and Narrow Scope

- Consider the sentence 'The present king of France is not bald'.
- This sentence is ambiguous.
- It may mean that it is false to say that the present king of France is bald, in which case the negation has **wide scope**, while the description has **narrow scope**.
  - $\sim(\exists!x)(Fx \,\&\, (\forall x)(Fx \supset Bx))$
- Or it may mean that the present king of France lacks the property being bald, in which case the negation has **narrow scope** while the description has **wide scope**.
  - $(\exists!x)(Fx \,\&\, (\forall x)(Fx \supset \sim Bx))$

Dealing with Ambiguity

- The narrow scope reading of the description is true if there is no present king of France, since '$(\exists!x)(Fx \,\&\, (\forall x)(Fx \supset Bx))$' is false.
- The wide scope reading of the description is false if there is no present king of France, since no object satisfies the condition specified by '$(Fx \,\&\, (\forall x)(Fx \supset \sim Bx))$'.
- The abbreviation of the first reading involves a convention that reflects the wide scope of the negation symbol:

- ∼[B(The x)Fx]

- The abbreviation of the second reading reflects the narrow scope of the negation symbol, which governs ‘Bx’ in the original sentence’.

  - ∼B(The x)Fx

- The text refers to the “negated predicate” ‘∼B,’ but negation is an operator applying to sentences, and so it is the open sentence ‘Bx’ that is negated in ‘(∃!x)(Fx & ∼Bx).’