

Name \_\_\_\_\_

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**Second Midterm**  
**Philosophy 112**  
**Winter 2005**

Answer the following questions in the spaces below them. You may use scratch paper.

1. Draw links to show which quantifiers bind which variables and say which occurrences of the variables are bound and which are free. 5 points.

$$(\overbrace{\forall x}^1)((\overbrace{Fx}^1 \equiv (\overbrace{\forall y}^{23})Lx y}) \vee (\overbrace{\exists x}^{45})Lyx).$$

1 Bound

2 Bound

3 Bound

4 Free

5 Bound

2. Write a substitution instance using 'c' for the following sentence. 5 points.

$$(\exists y)[Fy \ \& \ (\forall x)((Fx \supset Gxy) \ \& \ (\forall y)Fy)]$$

$$Fc \ \& \ (\forall x)((Fx \supset Gxc) \ \& \ (\forall y)Fy)$$

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3. Using the transcription guide on the last page of this exam, transcribe the following sentences of English into Predicate Logic, using unrestricted quantifiers only. 10 points each.

a. Every even number is greater than at least one odd number.

$$(\forall x)(Ex \supset (\exists y)(Oy \ \& \ Gxy))$$

b. If some number is greater than two, it is three.

$$(\forall x)(Gxd \supset x = t)$$

c. There are exactly two odd numbers.

$$(\exists x)(\exists y)((Ox \ \& \ Oy) \ \& \ (\forall z)(Oz \supset (x = z \vee y = z)))$$

4. Using the transcription guide on the last page of this exam, give alternative transcriptions which show the way the following sentence is ambiguous. 10 points.

All numbers greater than two are not even.

$$(\forall x)(Gxd \supset \sim Ex)$$

$$\sim(\forall x)(Gxd \supset Ex)$$

5. Using the transcription guide on the last page of this exam, transcribe the following sentences of English into Predicate Logic. Expressions of the form (The **u**) and  $(\exists! \mathbf{u})$  should not appear in your final answer. (Note, transcribe 'the even number' as a definite description, not as a name.) 10 points.

The even number is greater than at least one odd number.

$$(\exists x)((Ex \ \& \ (\forall y)(Ey \supset x = y)) \ \& \ (\exists z)(Oz \ \& \ Gxz))$$

6. Using the transcription guide on the last page of this exam, transcribe the following sentence of Predicate Logic into fluent English. 10 points.

$$(\forall x)(\forall y)(\forall z)[(Ox \ \& \ Oy \ \& \ Oz) \supset (x = y \vee y = z \vee x = z)].$$

There are at most two odd numbers.

7. Provide derivations which establish the validity of the following arguments. 10 points each.

a.

$$\frac{(\exists x)(\forall y)Lxy}{(\forall x)(\exists y)Lyx}$$

1	(∃x)(∀y)Lxy	P
2	a   (∀y)Lay	A
3	La <sup>∧</sup>	2 ∀E
4	(∃y)Ly <sup>∧</sup>	3 ∃I
5	(∀x)(∃y)Lyx	4 ∀I
6	(∀x)(∃y)Lyx	1 2-5 ∃E

b.

$$\frac{\begin{array}{l} (\forall x)(Bx \supset Gxf(x)) \\ (\forall x)Bf(x) \end{array}}{(\forall x)Gf(x)f(f(x))}$$

1	( $\forall x$ )( $Bx \supset Gxf(x)$ )	P
2	( $\forall x$ ) $Bf(x)$	P
3	$Bf(\hat{a})$	$2 \forall E$
4	$Bf(\hat{a}) \supset Gf(\hat{a})f(f(\hat{a}))$	$1 \forall E$
5	$Gf(\hat{a})f(f(\hat{a}))$	$3,4 \supset E$
6	( $\forall x$ ) $Gf(x)f(f(x))$	$5 \forall I$

c.

$$\frac{(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y) \ \& \ Bx)}{(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y)) \ \& \ (\forall x)(Fx \supset Bx)}$$

1	$(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y) \ \& \ Bx)$	P
2	$a \quad Fa \ \& \ (\forall y)(Fy \supset a = y) \ \& \ Ba$	A
3	$Fa$	2 & E
4	$(\forall y)(Fy \supset a = y)$	2 & E
5	$Ba$	2 & E
6	$Fb$	A
7	$Fb \supset a = b$	4 $\forall$ E
8	$a = b$	6,7 $\supset$ E
9	$Bb$	5,8 = E
10	$F\hat{b} \supset B\hat{b}$	6-9 $\supset$ I
11	$(\forall x)(Fx \supset Bx)$	10 $\forall$ I
12	$Fa \ \& \ (\forall y)(Fy \supset a = y)$	3, 11 & I
13	$(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y))$	12 $\exists$ I
14	$(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y)) \ \& \ (\forall x)(Fx \supset Bx)$	11, 13 & I
15	$(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y)) \ \& \ (\forall x)(Fx \supset Bx)$	1, 2-14 $\exists$ E

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Transcription guide for problems 3-5 (tear-off).

Domain: {x: x is a positive integer}

o: one

d: two

t: three

Ex: x is even

Ox: x is odd

Gxy: x is greater than y

$f(x,y)$ : the addition function,  $x + y$