First Midterm Solutions
Philosophy 112
Winter 2005

Answer the following questions in the spaces below them. You may use scratch paper.

1. For each of the underlined expressions, state whether the expression is being used as part of the metalanguage, mentioned as part of the metalanguage, used as part of the object language, or mentioned as part of the object language. 5 points each.

a. ⟨d[Eve/x](x), v(a)⟩ satisfies 'Lxa'.

Used as part of the metalanguage.

b. If '**s**' indicates the constant 'a', then 'Lea' can be represented as '**P(s)**'.

Mentioned as part of metalanguage.

2. Which of the following facts are semantical facts, and which are syntactical facts? 5 points each.

a. Premises: (∃x)Fx, (∃x)Gx; Conclusion: (∃x)(Fx & Gx).

Syntactical fact. (Explanation: Although the role of these sentences of Predicate Logic in an argument is specified, nothing is stated about how the sentences are interpreted or whether they are true, which is the nature of semantical facts.)

b. **d** satisfies 'Ga.'

Semantical fact. (Explanation: The metavariable '**d**' indicates a valuation function which is part of an interpretation. Moreover, the relation of satisfying a sentence is a semantical notion, since it is defined in terms of valuation functions.)

3. Using the transcription guide on the last page of this exam, transcribe the following sentences of Predicate Logic into colloquial English. 10 points each.

a. (∃x)(Ox & G*f*(x,o)t).

There is an odd positive integer which, when added to one, results in a sum that is greater than three.

b. (∀x)((Ox & ~Gxw) ⊃ x = o).

The only odd positive integer that is not greater than two is one.

Alternatively: One is the only odd positive integer that is not greater than two.

Alternatively: Any odd positive integer which is not greater than two is equal to one.

4. Using the transcription guide on the last page of this exam, transcribe the following sentences of English into Predicate Logic. 10 points each

a. The sum of one and three is greater than the sum of one and two.

G*f*(o,t)*f*(o,w)

b. No positive integer that is not odd is greater than 2.

Note: This is a false sentence. For example, the number 4 is not odd but is greater than 2.

~(∃x)(~Ox & Gxw)

Alternatively: (∀x)(~Ox ⊃ ~Gxw)

Alternatively: (∀x)(Gxw ⊃ Ox)

5. Using the transcription guide on the last page of this exam, determine the truth-values of the following sentences, and show how you arrived at them. 10 points each.

a. (∀x)Gf(x,o)x. (Use any method.)

This says that for any positive integer, the result of adding one to it is greater than it. That this is a true sentence can be seen from knowledge of elementry arithmetic. We can reason informally as follows. Suppose **u** is an arbitrary positive integer. Then the result of adding **u** and 1 is **u**+1. But **u** + 1 is greater than **u**, for any value of **u**. Therefore, the result of adding 1 to **u** is greater than **u** itself, and since this holds for all numbers **u**, the universally quantified sentence is true, QED.

b. ~(∃x)(Ox & ~Gxw). (Use formal semantics.)

This says that no odd positive integer is not greater than two. This is false, as can be seen from knowledge of elementary arithmetic. The number one is odd and it is not greater than two.

Let 'd' be an arbitrary variable assignment. d[1/x] satisfies 'Ox,' since $\langle 1 \rangle \in$ **v**(O). Moreover $\langle 1, 2 \rangle \notin$ **v**(G), and therefore ⟨ d[1/x](x),v(w)⟩ ∉ **v**(G), Thus, d[1/x] does not satisfy 'Gxw.' Therefore, d[1/x] satisfies '~Gxw.' We may conclude that d[1/x] satisfies 'Ox & ~Gxw.' So there is at least one x-variant of d which satisfies that sentence, in which case d itself satisfies '(∃x)(Ox & ~Gxw).' Then d does not satisfy '~(∃x)(Ox & ~Gxw).' Since the choice of d is arbitrary, no variable assignment satisfies '~(∃x)(Ox & ~Gxw),' in which case it is false, QED.

6. Evaluate the following arguments of Predicate Logic for validity. If the argument is invalid, give a counterexample in the formal semantics. If it is valid, show why it is so (using the formal semantics or not). 10 points each.

(∀x)(Fa ∨ Gx)

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(∀x)Gx ∨ Fa

This argument is valid. If everything is such that either a is F or it is G, then either everything is such that it is G or a is F.

Validity will be shown using formal semantics. Let **I** be an arbitrary interpretation. Suppose that '(∀x)(Fa ∨ Gx)' is true in **I**. Then for all variable assignments **d**, **d** satisfies '(∀x)(Fa ∨ Gx).' So all x-variants of **d** satisfy 'Fa ∨ Gx.' Let **d**[**u**/x] be an arbitrary such x-variant. Then either **d**[**u**/x] satisfies 'Fa' or **d**[**u**/x] satisifies 'Gx.' If **d**[**u**/x] satisfies 'Fa,' then **d**[**u**/x] satisfies '(∀x)Gx ∨ Fa.' If **d**[**u**/x] satisfies 'Gx,' then since the choice of **u** is arbitrary, all x-variants of **d** satisfy 'Gx,' in which case **d** satisfies '(∀x)Gx.' Then **d** satisfies '(∀x)Gx ∨ Fa' as well. On either of the two possible assumptions, **d** satisfies '(∀x)Gx ∨ Fa.' So **d** does satisfy '(∀x)Gx ∨ Fa.' Then '(∀x)Gx ∨ Fa' is true in **I**. Since the choice of **I** is arbitrary, on any interpretation, if '(∀x)(Fa ∨ Gx)' is true, then '(∀x)Gx ∨ Fa' is true, QED.

(∃x)Fx ⊃ (∃x)Gx)
Fa

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Ga

This argument is invalid. Take as the domain the positive integers. Fx: x is odd. Gx: x is prime. It is true that if there is an odd number, then there is a prime number. It is also true that 9 is an odd number, but it is not true that 9 is a prime number.

In terms of the formal semantics, let ⟨9⟩ ∈ v(F), ⟨3⟩ ∈ v(G), ⟨ 9⟩ ∉ v(G), and v(a) = 9. Therefore, for arbitrary d, d[9/x] satisfies 'Fx.' So there is an x-variant of d which satisfies 'Fx,' in which case d satisfies '(∃x)Fx.' Then that sentence is true. Further, since ⟨3⟩ ∈ v(G), for arbitrary d, d[3/x] satisfies 'Gx,' so that an x-variant of d satisfies 'Gx' and d itself satisfies '(∃x)Gx.' Putting these facts together, d satisifies (∃x)Fx ⊃ (∃x)Gx). Further, ⟨v(a)⟩ ∈ v(F), since ⟨9⟩ ∈ v(F). Therefore, 'Fa' is true. Hoewever, ⟨v(a)⟩ ∉ v(G), since ⟨9⟩ ∉ v(G). Therefore, 'Ga' is false, QED.

Transcription guide for problems 3-5 (tear-off).

Domain: {x: x is a positive integer}
o: one
w: two
t: three
Ox: x is odd
Gxy: x is greater than y
f(x,y): the addition function, x + y