Name ___________________________________

Final Examination
Philosophy 112
Winter 2005

Please work all the problems in the space provided. You may use only the techniques noted on the individual problems. Please be sure that you do everything that is asked for in each problem. Also, in each answer, bring out as much detail as possible.

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1. Transcribe the following argument, revealing as much structure as possible and providing a transcription guide, with a domain that includes everything. (15 points)

The sum of one and one is two.
If a positive integer is odd, then the sum of it and one is even.
One is odd.

Two is even.

D = Everything
Px: x is a positive integer, Ex: x is even, Ox: x is odd
s: addition function
o: one, w: two

s(o,o)=w
(∀x)((Px & Ox) ⊃ E*s*(x,o))
Oo
∴ Ew

Note: given this transcription, the argument is not formally valid. For it to be formally valid, the third premise would have to be something like 'One is an odd positive integer.' The argument would also be formally valid if the domain were restricted to positive integers, in which case the second premise would be transcribed as '(∀x)(Ox ⊃ E*s*(x,o)).'

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2. Using the formal semantics for Predicate Logic (**not a derivation**), show whether the following argument is valid or invalid. (15 points)

$(\exists x)(Mx \;\&\; Px)$
$(\forall x)(Ix \supset Mx)$
―――――――――――
$(\exists x)(Ix \;\&\; Px)$

The argument is invalid.
D = All animals
Ix: x is human, Mx, x is a mammal, Px, x is furry

Let **I** be an interpretation and **d** be a variable assignment based on **I**. Suppose a given object **u** in D is in the extension of 'I': v(**u**) $\in$ v(I). Then **d**[**u**/x] satisifies 'Ix.' Moreover, any such object is in the extension of 'M' as well, since all humans are mammals: v(**u**) $\in$ v(M). So **d**[**u**/x] satisifies 'Mx.' Therefore, if **d**[**u**/x] satisfies 'Ix,' then it satisfies 'Mx,' in which case **d**[**u**/x] satisfies 'Ix $\supset$ Mx.' Since the choice of **u** is arbitrary, **d** satisfies '$(\forall x)(Ix \supset Mx)$.' Thus, the sentence is true in **I**.

Let **d** be a variable assignment based on **I**. Let **u** be a member of D which is a mouse. Then v(**u**) $\in$ v(M), in which case **d**[**u**/x] satisfies 'Mx.' Mice are furry, so v(**u**) $\in$ v(I), in which case **d**[**u**/x] satisfies 'Ix.' Then **d**[**u**/x] satisfies 'Mx & Px.' Since there is an x-variant of **d** which satisfies 'Mx & Px,' **d** satisfies '$(\exists x)(Mx \;\&\; Px)$,' and the sentence is true in **I**.

However, there is no member of **u** which is in the extension of both 'I' and 'P,' since humans lack fur. Hence there is no x-variant of any variable assignment **d** which satisfies both 'Ix' and 'Px.' Therefore, there is no such x-variant that satisfies 'Ix & Px.' So there is no variable assignment **d** which satisfies '$(\exists x)(Ix \;\&\; Px)$,' and the sentence is false in I. Therefore, the premises are true in **I** but the conclusion is false in **I**, and the argument is invalid.

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3. Using the formal semantics, show that the following two sentences of Predicate Logic are logically equivalent. (15 points)

(∀x)Cx
∼(∃x)∼ Cx

Let **I** be and arbitrary interpretation. '(∀x)Cx' is true in **I** if and only if it is satisfied by all variable assignments **d**. This holds if and only if **d**[**u**/x] satisfies 'Cx' for all members **u** in the domain of **I**. This holds if and only if for all members **u** in the domain of **I**, **d**[**u**/x] does not satisfy '∼Cx.' This holds if and only if **d** does not satisfy '(∃x)∼Cx.' This holds if and only if **d** satisfies '∼(∃x)∼Cx.' Finally, this holds if and only if '∼(∃x)∼Cx' is true in **I**. So in an arbitrary interpretation, '(∀x)Cx' is true if and only if '∼(∃x)∼Cx' is true. In that case, the two sentences are logically equivalent.

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4. Show that the following sentence is neither logically true nor a contradiction by constructing an interpretation on which it is true and one on which it is false. State why it is true in the interpretation in which it is true, and why it is false in the interpretation on which it is false. (15 points)

(∃x)(Fx & (∀y)(Fy ⊃ x = y)) & (∀x)(Fx ⊃ Gx)

For both interpretations, let the domain be persons.

The sentence is true if the predicate ‘F’ has as its extension the set of current Presidents of the United States, and the predicate ‘G’ has as its extension the set of current Commanders-in-Chief of the United States armed forces. for the first conjunct, there is a current President and there is only one current President: any current President is identical to him. So the first conjuct is true. For the second conjunct, any President is Commander-in-Chief, so any object in the extension of ‘F’ is in the extension of ‘G,’ in which case the second conjunct is true as well, and the whole conjunction is true.

To make the sentence false, one need only make one of the conjuncts false. For convenience, we will continue to let the extension of ‘F’ be the set of current Presidents. Then the first conjunct is true, as before. But we will let the extension of ‘G’ be the set of all women. Then the right conjunct would assert that any object that is the President is a woman, which is false, and so the conjunction is false.

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5. Determine the truth-value of the following sentence in the interpretation given. Show in detail how the truth-value is determined. (10 points)

(∀z)(Ez ≡ ((∃y)Gzy & (∃x)Gxz))

Domain: {1, 2, 3}
Ex: x is even
Gxy: x is greater than y

This sentence can be transcribed into English as follows: An integer from 1 to 3 is even if and only if there is such an integer that it is greater than and there is such an integer that is greater than it.

The sentence is true. First, note that the number 2 is the only even number in the domain. Further, there is a number greater than 2 in the domain, i.e. the number 3, and there is a number than which 2 is greater, i.e., the number 1.

So if a number is even, it is two, and given that it is two, then some number is greater than it, and some number is less than it.

Second, note that 2 is the only number than which some number is greater and which is greater than some number. So, if there is a number greater than a given number and the latter number is greater than some number, the number is 2. But 2 is even. So if there is a number greater than a given number and the latter number is greater than some number, that number is even.

We may conclude that any number is such that it is even if and only if there is a number greater than it and it is greater than some number.

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6. Prove that the following collection of sentences is inconsistent by providing a derivation. Quantifier Negation rules may be used. (15 points)

$(\forall x)Fx \vee (\forall x){\sim}Fx$, $(\exists x)Fx \equiv (\exists x){\sim}Fx$

1	$(\forall x)Fx \vee (\forall x) \sim Fx$	P
2	$(\exists x)Fx \equiv (\exists x) \sim Fx$	P
3	$\quad (\forall x)Fx$	A
4	$\quad F\hat{a}$	$3 \forall E$
5	$\quad (\exists x)Fx$	$4 \exists I$
6	$\quad (\exists x) \sim Fx$	$2, 5 \equiv E$
7	$\quad \sim (\forall x)Fx$	$7 \exists \sim$
8	$\quad \bot$	$3, 7 \bot I$
9	$\quad (\forall x) \sim Fx$	A
10	$\quad \sim F\hat{a}$	$9 \forall E$
11	$\quad (\exists x) \sim Fx$	$10 \exists I$
12	$\quad (\exists x)Fx$	$2, 11 \equiv E$
13	$\quad \sim (\exists x)Fx$	$9 \forall \sim$
14	$\quad \bot$	$12\ 13 \bot I$
15	$\bot$	1 3-8, 9-14 AC

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7. Prove that the following sentence is a contradiction by providing a derivation, **without using the Quantifier Negation rules**. (15 points)

[(∀x)Fx ⊃ (∃y)Gy] & [∼(∃x)Gx & ∼(∃x)∼Fx]

1	[(∀x)Fx ⊃ (∃y)Gy] & [∼ (∃x)Gx & ∼ (∃x) ∼ Fx]	P
2	(∀x)Fx ⊃ (∃y)Gy	1 & E
3	∼ (∃x)Gx & ∼ (∃x) ∼ Fx	1 & E
4	∼ (∃x)Gx	3 & E
5	∼ (∃x) ∼ Fx	3 & E
6	∼ (∀x)Fx	2,4 DC
7	| ∼ Fa	A
8	| (∃x) ∼ Fx	7 ∃ I
9	| ∼ (∃x) ∼ Fx	5 R
10	Fâ	7-9 RD
11	(∀x)Fx	10 ∀ I