

*Principles of Mathematics*  
Chapter IV  
PROPER NAMES, ADJECTIVES, AND  
VERBS

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46. In the present chapter, certain questions are to be discussed belonging to what may be called philosophical grammar. The study of grammar, in my opinion, is capable of throwing far more light on philosophical questions than is commonly supposed by philosophers. Although a grammatical distinction cannot be uncritically assumed to correspond to a genuine philosophical difference, yet the one is *prima facie* evidence of the other, and may often be most usefully employed as a source of discovery. Moreover, it must be admitted, I think, that every word occurring in a sentence must have *some* meaning: a perfectly meaningless sound could not be employed in the more or less fixed way in which language employs words. The correctness of our philosophical analysis of a proposition may therefore be usefully checked by the exercise of assigning the meaning of each word in the sentence expressing the proposition. On the whole, grammar seems to me to bring us much nearer to a correct logic than the current opinions of philosophers; and in what follows, grammar, though not our master, will yet be taken as our guide. [Note: The excellence of grammar as a guide is proportional to the paucity of inflexions, i.e. to the degree of analysis effected by the language considered.]

Of the parts of speech, three are specially important: substantives, adjectives, and verbs. Among substantives, some are derived from adjectives or verbs, as humanity from human, or sequence from follows. (I am not speaking of an etymological derivation, but of a logical one.) Others, such as proper names, or space, time, and matter, are not derivative, but appear primarily as substantives. What we wish to obtain is a classification, not of words, but of ideas; I shall therefore call adjectives or predicates all notions which are capable of being such, even in a form in which grammar would call them substantives. The fact is, as we shall see, that *human* and *humanity* denote precisely the same concept, these words being employed respectively according to the kind of relation in which this concept stands to the other constituents of a proposition in which it occurs. The distinction which we require [p. 43] is not identical with the grammatical distinction between substantive and adjective, since one

single concept may, according to circumstances, be either substantive or adjective: it is the distinction between proper and general names that we require, or rather between the objects indicated by such names. In every proposition, as we saw in Chapter III, we may make an analysis into something asserted and something about which the assertion is made. A proper name, when it occurs in a proposition, is always, at least according to one of the possible ways of analysis (where there are several), the subject that the proposition or some subordinate constituent proposition is about, and not what is said about the subject. Adjectives and verbs, on the other hand, are capable of occurring in propositions in which they cannot be regarded as subject, but only as parts of the assertion. Adjectives are distinguished by capacity for *denoting*—a term which I intend to use in a technical sense to be discussed in Chapter V. Verbs are distinguished by a special kind of connection, exceedingly hard to define, with truth and falsehood, in virtue of which they distinguish an asserted proposition from an unasserted one, *e.g.* “Caesar died” from “the death of Caesar.” These distinctions must now be amplified, and I shall begin with the distinction between general and proper names.

47. Philosophy is familiar with a certain set of distinctions, all more or less equivalent: I mean, the distinctions of subject and predicate, substance and attribute, substantive and adjective, *this* and *what*. [Note: This last pair of terms is due to Mr Bradley.] I wish now to point out briefly what appears to me to be the truth concerning these cognate distinctions. The subject is important, since the issues between monism and monadism, between idealism and empiricism, and between those who maintain and those who deny that all truth is concerned with what exists, all depend, in whole or in part, upon the theory we adopt in regard to the present question. But the subject is treated here only because it is essential to any doctrine of number or of the nature of the variable. Its bearings on general philosophy, important as they are, will be left wholly out of account.

Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity. The first two emphasize the fact that every term is *one*, while the third is derived from the fact that every term has being, *i.e.* *is* in some sense. A man, a moment, a number, a class, a relation, a chimaera, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false.

It might perhaps be thought that a word of such extreme generality could not be of any great use. Such a view, however, owing to certain [p. 44] wide-spread philosophical doctrines, would be erroneous. A term is, in fact, possessed of all the properties commonly assigned to substances, or substantives. Every term, to begin with, is a logical subject: it is, for example, the subject of the proposition that itself is one. Again every term is immutable and indestructible. What a term is, it is, and no change can be conceived in it which would not destroy its identity and make it another term. [Note: The notion of a term here set forth is a modification of Mr G. E. Moore’s notion of a concept in his article “On the

Nature of Judgment," *Mind*, N. S. No. 30, from which notion, however, it differs in some important respects.] Another mark which belongs to terms is numerical identity with themselves and numerical diversity from all other terms. [Note: On identity, see Mr G. E. Moore's article in the *Proceedings of the Aristotelian Society*, 1900-1901.] Numerical identity and diversity are the source of unity and plurality; and thus the admission of many terms destroys monism. And it seems undeniable that every constituent of every proposition can be counted as one, and that no proposition contains less than two constituents. *Term* is, therefore, a useful word, since it marks dissent from various philosophies, as well as because, in many statements, we wish to speak of *any* term or *some* term.

48. Among terms, it is possible to distinguish two kinds, which I shall call respectively *things* and *concepts*. The former are the terms indicated by proper names, the latter those indicated by all other words. Here proper names are to be understood in a somewhat wider sense than is usual, and things also are to be understood as embracing all particular points and instants, and many other entities not commonly called things. Among concepts, again, two kinds at least must be distinguished, namely those indicated by adjectives and those indicated by verbs. The former kind will often be called predicates or class-concepts; the latter are always or almost always relations. (In intransitive verbs, the notion expressed by the verb is complex, and usually asserts a definite relation to an indefinite relatum, as in "Smith breathes.")

In a large class of propositions, we agreed, it is possible, in one or more ways, to distinguish a subject and an assertion about the subject. The assertion must always contain a verb, but except in this respect, assertions appear to have no universal properties. In a relational proposition, say "*A* is greater than *B*," we may regard *A* as the subject, and "is greater than *B*" as the assertion, or *B* as the subject and "*A* is greater than" as the assertion. There are thus, in the case proposed, two ways of analyzing the proposition into subject and assertion. Where a relation has more than two terms, as in "*A* is here now," [Note: This proposition means "*A* is in this place at this time." It will be shown in Part VII that the relation expressed is not reducible to a two-term relation.] there will be more than two ways of making the analysis. But in some propositions, there is only a single way. These are the subject- [p.45] predicate propositions, such as "Socrates is human." The proposition "humanity belongs to Socrates," which is equivalent to "Socrates is human," is an assertion about humanity; but it is a distinct proposition. In "Socrates is human," the notion expressed by *human* occurs in a different way from that in which it occurs when it is called *humanity*, the difference being that in the latter case, but not in the former, the proposition is *about* this notion. This indicates that humanity is a concept, not a thing. I shall speak of the *terms* of a proposition as those terms, however numerous, which occur in a proposition and may be regarded as subjects about which the proposition is. It is a characteristic of the terms of a proposition that any one of them may be replaced by any other entity without our ceasing to have a proposition. Thus we shall say that "Socrates is human" is a proposition having only one term; of the remaining components of the proposition, one is the verb, the other is a *predicate*. With the sense which

*is* has in this proposition, we no longer have a proposition at all if we replace *human* by something other than a predicate. Predicates, then, are concepts, other than verbs, which occur in propositions having only one term or subject. Socrates is a thing, because Socrates can never occur otherwise than as term in a proposition: Socrates is not capable of that curious twofold use which is involved in *human* and *humanity*. Points, instants, bits of matter, particular states of mind, and particular existents generally, are things in the above sense, and so are many terms which do not exist, for example, the points in a non-Euclidean space and the pseudo-existents of a novel. All classes, it would seem, as numbers, men, spaces, etc., when taken as single terms, are things; but this is a point for Chapter VI.

Predicates are distinguished from other terms by a number of very interesting properties, chief among which is their connection with what I shall call *denoting*. One predicate always gives rise to a host of cognate notions: thus in addition to *human* and *humanity*, which only differ grammatically, we have *man*, *a man*, *some man*, *any man*, *every man*, *all men* [Note: I use *all men* as collective, *i.e.* as nearly synonymous with the human race, but differing therefrom by being many and not one. I shall always use *all* collectively, confining myself to *every* for the distributive sense. Thus I shall say “every man is mortal,” not “all men are mortal.”], all of which appear to be genuinely distinct one from another. The study of these various notions is absolutely vital to any philosophy of mathematics; and it is on account of them that the theory of predicates is important.

49. It might be thought that a distinction ought to be made between a concept as such and a concept used as a term, between, *e.g.*, such pairs as *is* and *being*, *human* and *humanity*, *one* in such a proposition as “this is one” and 1 in “1 is a number.” But inextricable difficulties will envelop us if we allow such a view. There is, [p.46] of course, a grammatical difference, and this corresponds to a difference as regards relations. In the first case, the concept in question is used as a concept, that is, it is actually predicated of a term or asserted to relate two or more terms; while in the second case, the concept is itself said to have a predicate or a relation. There is, therefore, no difficulty in accounting for the grammatical difference. But what I wish to urge is, that the difference lies solely in external relations, and not in the intrinsic nature of the terms. For suppose that *one* as adjective differed from 1 as term. In this statement, *one* as adjective has been made into a term; hence either it has become 1, in which case the supposition is self-contradictory; or there is some other difference between *one* and 1 in addition to the fact that the first denotes a concept not a term while the second denotes a concept which is a term. But in this latter hypothesis, there must be propositions concerning *one* as term, and we shall still have to maintain propositions concerning *one* as adjective as opposed to *one* as term; yet all such propositions must be false, since a proposition about *one* as adjective makes *one* the subject, and is therefore really about *one* as term. In short, if there were any adjectives which could not be made into substantives without change of meaning, all propositions concerning such adjectives (since they would necessarily turn them into substantives) would be false, and so would

the proposition that all such propositions are false, since this itself turns the adjectives into substantives. But this state of things is self-contradictory.

The above argument proves that we were right in saying that terms embrace everything that can occur in a proposition, with the possible exception of complexes of terms of the kind denoted by *any* and cognate words [Note: See the next chapter]. For if *A* occurs in a proposition, then, in this statement, *A* is the subject; and we have just seen that, if *A* is ever not the subject, it is exactly and numerically the same *A* which is not subject in one proposition and is subject in another. Thus the theory that there are adjectives or attributes or ideal things, or whatever they may be called, which are in some way less substantial, less self-subsistent, less self-identical, than true substantives, appears to be wholly erroneous, and to be easily reduced to a contradiction. Terms which are concepts differ from those which are not, not in respect of self-subsistence, but in virtue of the fact that, in certain true or false propositions, they occur in a manner which is different in an indefinable way from the manner in which subjects or terms of relations occur.

50. Two concepts have, in addition to the numerical diversity which belongs to them as terms, another special kind of diversity which may be called conceptual. This may be characterized by the fact that two propositions in which the concepts occur otherwise than as terms, even if, in all other respects, the two propositions are identical, [p. 47] yet differ in virtue of the fact that the concepts which occur in them are conceptually diverse. Conceptual diversity implies numerical diversity, but the converse implication does not hold, since not all terms are concepts. Numerical diversity, as its name implies, is the source of plurality, and conceptual diversity is less important to mathematics. But the whole possibility of making different assertions about a given term or set of terms depends upon conceptual diversity, which is therefore fundamental in general logic.

51. It is interesting and not unimportant to examine very briefly the connection of the above doctrine of adjectives with certain traditional views on the nature of propositions. It is customary to regard all propositions as having a subject and a predicate, *i.e.* as having an immediate *this*, and a general concept attached to it by way of description. This is, of course, an account of the theory in question which will strike its adherents as extremely crude; but it will serve for a general indication of the view to be discussed. This doctrine develops by internal logical necessity into the theory of Mr Bradley's Logic, that all words stand for ideas having what he calls *meaning*, and that in every judgment there is a something, the true subject of the judgment, which is not an idea and does not have meaning. To have meaning, it seems to me, is a notion confusedly compounded of logical and psychological elements. *Words* all have meaning, in the simple sense that they are symbols which stand for something other than themselves. But a proposition, unless it happens to be linguistic, does not itself contain words: it contains the entities indicated by words. Thus meaning, in the sense in which words have meaning, is irrelevant to logic. But such concepts as *a man* have meaning in another sense: they are, so to speak, symbolic in their own logical nature, because they have the property which I call *denoting*.

That is to say, when a man occurs in a proposition (*e.g.* “I met a man in the street”), the proposition is not about the concept *a man*, but about something quite different, some actual biped denoted by the concept. Thus concepts of this kind have meaning in a non-psychological sense. And in this sense, when we say “this is a man,” we are making a proposition in which a concept is in some sense attached to what is not a concept. But when meaning is thus understood, the entity indicated by *John* does not have meaning, as Mr Bradley contends [Note: *Logic*, Book I, Chap. II, secs. 17, 18 (pp. 58-60).]; and even among concepts, it is only those that denote that have meaning. The confusion is largely due, I believe, to the notion that *words* occur in propositions, which in turn is due to the notion that propositions are essentially mental and are to be identified with cognitions. But these topics of general philosophy must be pursued no further in this work.

52. It remains to discuss the verb, and to find marks by which it is distinguished from the adjective. In regard to verbs also, there is [p. 48] a twofold grammatical form corresponding to a difference in merely external relations. There is the verb in the form which it has as verb (the various inflexions of this form may be left out of account), and there is the verbal noun, indicated by the infinitive or (in English) the present participle. The distinction is that between “Felton killed Buckingham” and “Killing no murder.” By analyzing this difference, the nature and function of the verb will appear.

It is plain, to begin with, that the concept which occurs in the verbal noun is the very same as that which occurs as verb. This results from the previous argument, that every constituent of every proposition must, on pain of self-contradiction, be capable of being made a logical subject. If we say “*kills* does not mean the same as *to kill*,” we have already made *kills* a subject, and we cannot say that the concept expressed by the word *kills* cannot be made a subject. Thus the very verb which occurs as verb can occur also as subject. The question is: What logical difference is expressed by the difference of grammatical form? And it is plain that the difference must be one in external relations. But in regard to verbs, there is a further point. By transforming the verb, as it occurs in a proposition, into a verbal noun, the whole proposition can be turned into a single logical subject, no longer asserted, and no longer containing in itself truth or falsehood. But here too, there seems to be no possibility of maintaining that the logical subject which results is a different entity from the proposition. “Caesar died” and “the death of Caesar” will illustrate this point. If we ask: What is asserted in the proposition “Caesar died”? the answer must be “the death of Caesar is asserted.” In that case, it would seem, it is the death of Caesar which is true or false; and yet neither truth nor falsity belongs to a mere logical subject. The answer here seems to be that the death of Caesar has an external relation to truth or falsehood (as the case may be), whereas “Caesar died” in some way or other contains its own truth or falsehood as an element. But if this is the correct analysis, it is difficult to see how “Caesar died” differs from “the truth of Caesar’s death” in the case where it is true, or “the falsehood of Caesar’s death” in the other case. Yet it is quite plain that the latter, at any rate, is never equivalent to “Caesar died.” There appears to be an ultimate

notion of assertion, given by the verb, which is lost as soon as we substitute a verbal noun, and is lost when the proposition in question is made the subject of some other proposition. This does not depend upon grammatical form; for if I say “*Caesar died* is a proposition,” I do not assert that Caesar did die, and an element which is present in “Caesar died” has disappeared. Thus the contradiction which was to have been avoided, of an entity which cannot be made a logical subject, appears to have here become inevitable. This difficulty, which seems to be inherent in the very nature of truth and falsehood, is one with which I do not know how to deal satisfactorily. The most obvious course [p. 49] would be to say that the difference between an asserted and an unasserted proposition is not logical, but psychological. In the sense in which false propositions may be asserted, this is doubtless true. But there is another sense of assertion, very difficult to bring clearly before the mind, and yet quite undeniable, in which only true propositions are asserted. True and false propositions alike are in some sense entities, and are in some sense capable of being logical subjects; but when a proposition happens to be true, it has a further quality, over and above that which it shares with false propositions, and it is this further quality which is what I mean by assertion in a logical as opposed to a psychological sense. The nature of truth, however, belongs no more to the principles of mathematics than to the principles of everything else. I therefore leave this question to the logicians with the above brief indication of a difficulty.

53. It may be asked whether everything that, in the logical sense we are concerned with, is a verb, expresses a relation or not. It seems plain that, if we were right in holding that “Socrates is human” is a proposition having only one term, the *is* in this proposition cannot express a relation in the ordinary sense. In fact, subject-predicate propositions are distinguished by just this non-relational character. Nevertheless, a relation between Socrates and humanity is certainly *implied*, and it is very difficult to conceive the proposition as expressing no relation at all. We may perhaps say that it is a relation, although it is distinguished from other relations in that it does not permit itself to be regarded as an assertion concerning either of its terms indifferently, but only as an assertion concerning the referent. A similar remark may apply to the proposition “*A is*,” which holds of every term without exception. The *is* here is quite different from the *is* in “Socrates is human”; it may be regarded as complex, and as really predicating Being of *A*. In this way, the true logical verb in a proposition may be always regarded as asserting a relation. But it is so hard to know exactly what is meant by *relation* that the whole question is in danger of becoming purely verbal.

54. The twofold nature of the verb, as actual verb and as verbal noun, may be expressed, if all verbs are held to be relations, as the difference between a relation in itself and a relation actually relating. Consider, for example, the proposition “*A differs from B*.” The constituents of this proposition, if we analyze it, appear to be only *A*, difference, *B*. Yet these constituents, thus placed side by side, do not reconstitute the proposition. The difference which occurs in the proposition actually relates *A* and *B*, whereas the difference after analysis is a notion which has no connection with *A* and *B*. It may be said that we ought, in the analysis,

to mention the relations which difference has to  $A$  and  $B$ , relations which are expressed by *is* and *from* when we say “ $A$  is different from  $B$ .” These relations consist in the [p. 50] fact that  $A$  is referent and  $B$  relatum with respect to difference. But “ $A$ , referent, difference, relatum,  $B$ ” is still merely a list of terms, not a proposition. A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition. The verb, when used as a verb, embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term, though I do not know how to give a clear account of the precise nature of the distinction.

55. It may be doubted whether the general concept *difference* occurs at all in the proposition “ $A$  differs from  $B$ ,” or whether there is not rather a specific difference of  $A$  and  $B$ , and another specific difference of  $C$  and  $D$ , which are respectively affirmed in “ $A$  differs from  $B$ ” and “ $C$  differs from  $D$ .” In this way, difference becomes a class-concept of which there are as many instances as there are pairs of different terms; and the instances may be said, in Platonic phrase, to partake of the nature of difference. As this point is quite vital in the theory of relations, it may be well to dwell upon it. And first of all, I must point out that in “ $A$  differs from  $B$ ” I intend to consider the bare numerical difference in virtue of which they are two, not difference in this or that respect.

Let us first try the hypothesis that *a* difference is a complex notion, compounded of difference together with some special quality distinguishing a particular difference from every other particular difference. So far as the relation of difference itself is concerned, we are to suppose that no distinction can be made between different cases; but there are to be different associated qualities in different cases. But since cases are distinguished by their terms, the quality must be primarily associated with the terms, not with difference. If the quality be not a relation, it can have no special connection with the difference of  $A$  and  $B$ , which it was to render distinguishable from bare difference, and if it fails in this it becomes irrelevant. On the other hand, if it be a new relation between  $A$  and  $B$ , over and above difference, we shall have to hold that any two terms have two relations, difference and a specific difference, the latter not holding between any other pair of terms. This view is a combination of two others, of which the first holds that the abstract general relation of difference itself holds between  $A$  and  $B$ , while the second holds that when two terms differ they have, corresponding to this fact, a specific relation of difference, unique and unanalyzable and not shared by any other pair of terms. Either of these views may be held with either the denial or the affirmation of the other. Let us see what is to be said for and against them.

Against the notion of specific differences, it may be urged that, if differences differ, their differences from each other must also differ, and thus we are led into an endless process. Those who object to endless processes will see in this a proof that differences do not differ. But in [p.51] the present work, it will be maintained that there are no contradictions peculiar to the notion of infinity, and that an endless process is not to be objected to unless it arises in the analysis of the actual meaning of a proposition. In the present case, the process is one

of implications, not one of analysis; it must therefore be regarded as harmless.

Against the notion that the abstract relation of difference holds between  $A$  and  $B$ , we have the argument derived from the analysis of “ $A$  differs from  $B$ ” which gave rise to the present discussion. It is to be observed that the hypothesis which combines the general and the specific difference must suppose that there are two distinct propositions, the one affirming the general, the other the specific difference. Thus if there cannot be a general difference between  $A$  and  $B$ , this mediating hypothesis is also impossible. And we saw that the attempt to avoid the failure of analysis by including in the *meaning* of “ $A$  differs from  $B$ ” the relations of difference to  $A$  and  $B$  was vain. This attempt, in fact, leads to an endless process of the inadmissible kind; for we shall have to include the relations of the said relations to  $A$  and  $B$  and difference, and so on, and in this continually increasing complexity we are supposed to be only analyzing the *meaning* of our original proposition. This argument establishes a point of very great importance, namely, that when a relation holds between two terms, the relations of the relation to the terms, and of these relations to the relation and the terms, and so on *ad infinitum*, though all implied by the proposition affirming the original relation, form no part of the meaning of this proposition.

But the above argument does not suffice to prove that the relation of  $A$  to  $B$  cannot be abstract difference: it remains tenable that, as was suggested to begin with, the true solution lies in regarding every proposition as having a kind of unity which analysis cannot preserve, and which is lost even though it be mentioned by analysis as an element in the proposition. This view has doubtless its own difficulties, but the view that no two pairs of terms can have the same relation both contains difficulties of its own and fails to solve the difficulty for the sake of which it was invented. For, even if the difference of  $A$  and  $B$  be absolutely peculiar to  $A$  and  $B$ , still the three terms  $A$ ,  $B$ , difference of  $A$  from  $B$ , do not reconstitute the proposition “ $A$  differs from  $B$ ,” any more than  $A$  and  $B$  and difference did. And it seems plain that, even if differences did differ, they would still have to have something in common. But the most general way in which two terms can have something in common is by both having a given relation to a given term. Hence if no two pairs of terms can have the same relation, it follows that no two terms can have anything in common, and hence different differences will not be in any definable sense *instances* of difference [Note: The above argument appears to prove that Mr Moore’s theory of universals with numerically diverse instances in his paper on Identity (*Proceedings of the Aristotelian Society*, 1900-1901) must not be applied to all concepts. The relation of an instance to its universal, at any rate, must be actually and numerically the same in all cases where it occurs.] I conclude, then, that [p. 52] the relation affirmed between  $A$  and  $B$  in the proposition “ $A$  differs from  $B$ ” is the general relation of difference, and is precisely and numerically the same as the relation affirmed between  $C$  and  $D$  in “ $C$  differs from  $D$ .” And this doctrine must be held, for the same reasons, to be true of all other relations; relations do not have instances, but are strictly the same in all propositions in which they occur.

We may now sum up the main points elicited in our discussion of the verb.

The verb, we saw, is a concept which, like the adjective, may occur in a proposition without being one of the terms of the proposition, though it may also be made into a logical subject. One verb, and one only, must occur as verb in every proposition; but every proposition, by turning its verb into a verbal noun, can be changed into a single logical subject, of a kind which I shall call in future a propositional concept. Every verb, in the logical sense of the word, may be regarded as a relation; when it occurs as verb, it actually relates, but when it occurs as verbal noun it is the bare relation considered independently of the terms which it relates. Verbs do not, like adjectives, have instances, but are identical in all the cases of their occurrence. Owing to the way in which the verb actually relates the terms of a proposition, every proposition has a unity which renders it distinct from the sum of its constituents. All these points lead to logical problems, which, in a treatise on logic, would deserve to be fully and thoroughly discussed.

Having now given a general sketch of the nature of verbs and adjectives, I shall proceed, in the next two chapters, to discussions arising out of the consideration of adjectives, and in Chapter VII to topics connected with verbs. Broadly speaking, classes are connected with adjectives, while propositional functions involve verbs. It is for this reason that it has been necessary to deal at such length with a subject which might seem, at first sight, to be somewhat remote from the principles of mathematics.